Erratum to the paper "Metaconfluence of Calculi with Explicit Substitutions at a Distance"

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In [1], we proved metaconfluence of three calculi with explicit substitutions without including equational axioms. Nevertheless, a correction in the proof of the case of the structural λ-calculus is necessary. Our approach is based on the Hindley-Milner Theorem, where we assume that the following rewriting system do not have critical pairs:

$$\boxed{\begin{array}{lll l} t[x/u] & \mapsto_{\mathtt{c}} & t_{\langle x|y\rangle}[x/u][y/u] & \text{if } |t|_x > 1 \\ t[x/u] & \mapsto_{\mathtt{d}} & t\{x/u\} & \text{if } |t|_x = 1 \\ t[x/u] & \mapsto_{\mathtt{w}} & t & \text{if } |t|_x = 0 \end{array}}$$

However, the non-determinism of the rule $\mapsto_{\mathtt{c}}$ can lead to non-confluent terms. For instance, consider the following divergence:

$$\begin{tikzcd} & (X_y\ X_y)[y/a] \arrow[dl, "\mathtt{c}"'] \arrow[dr, "\mathtt{c}"] & \\ (X_z\ X_y)[y/a][z/a] & & (X_y\ X_z)[y/a][z/a] \end{tikzcd}$$

Note that the terms $(X_z\ X_y)[y/a][z/a]$ and $(X_y\ X_z)[y/a][z/a]$ are both in normal form. In order to close this diagram, one proceed as follows: $(X_z\ X_y)[y/a][z/a] =_\alpha (X_y\ X_z)[z/a][y/a] \equiv (X_y\ X_z)[y/a][z/a]$, where $\equiv$ is an axiom for commutation of independent substitutions. A detailed proof using this approach can be found at [2].

Acknowledgement

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References

[1] F. L. C. de Moura, D. Kesner, and M. Ayala-Rincón. Metaconfluence of Calculi with Explicit Substitutions at a Distance. In Venkatesh Raman and S. P. Suresh, editors, *34th International Conference on Foundation of Software Technology and Theoretical Computer Science (FSTTCS 2014)*, volume 29 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 391–402, Dagstuhl, Germany, 2014. Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik.

[2] F. Renaud. *Les Ressources Explicites vues par la Théorie de la Réécriture*. PhD thesis, Université Paris Diderot - Paris 7, 2011.